Least-Resistance Consensus: applying *Via Negativa* to decision-making *

Aleksander Kampa
ak@sikoba.com

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Abstract

This paper introduces Least-Resistance Consensus (LRC), a cardinal voting system that focuses entirely on measuring resistance. It is based on the observation that our "negative gut feeling" is often a strong, reliable indicator and that our dislikes are stronger than our likes. Measuring resistance rather than acceptance is therefore likely to provide a more accurate gauge when a choice between multiple options has to be made.

But LRC is not just a voting system. It encourages an iterative process of consensus-seeking in which possible options are proposed, discussed, amended and then either discarded or kept for a further round of voting.

LRC also proposes to take the structure of group opinion into account, with a view not only to minimizing overall group resistance but also to avoiding, as much as possible, there being sizeable groups of strong dissenters.

1 Introduction

In Western societies, the opinion that parliamentary democracy based on majority rule is the best possible form of government is widely held. We do know that minority opinions are not always represented and, as is the case in the UK with its first-past-the-post system, a party can gain an absolute majority with far fewer than half of the votes, but this seems to be considered a minor issue. Not just in government but at all levels of society, majority voting holds sway.

Members of tribal societies in Africa and elsewhere, who are used to the lengthy and complex processes needed for reaching a wide consensus among community members, would probably call the idea of majority voting primitive. It might even be difficult to convince some of them that such a patently absurd concept is actually used by humans.

Majority voting is, by its very nature, an antagonistic process. It is about one group winning a debate by sheer force of numbers: a more civilized version of "might is right". There are, of course,

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alternatives to majority voting that offer more nuanced procedures, but most are influenced by the concept of majority voting. Also, there is always the underlying and unspoken positive bias: the focus is on identifying the best solution. It is the positive, the affirmative, that underpins most of Western thinking.

In this paper, we will present the *Least-Resistance Consensus* (LRC) method, which focuses on determining which solution encounters the least resistance from a group. To begin with, however, we present a brief historical perspective of voting research.

2 Traditional Voting Systems

2.1 Some famous names in voting theory

The question of fair voting is probably as old as humanity. Plato proposed a multi-stage voting process more than 2,000 years ago, and Pliny the Younger wrote about strategic voting in the Roman Senate around 100 AD, as noted in Szpiro [1]. Many famous people have worked on election methods: the Medieval polymath and mystic Ramon Llull, the philosopher and cardinal Nicolaus Cusanus, the scientist Pierre-Simon Laplace and the Reverend Charles Dodgson, better known by his pen name, Lewis Carroll.

Two other names that are associated with the history of voting systems are the Marquis de Condorcet and Jean-Charles de Borda. They were contemporaries, and, indeed, the two traditional election systems are associated with their names.

2.2 The Condorcet and Borda methods

Apart from simple majority-voting, the limitations of which have been recognised since Antiquity, two classical voting methods have emerged.

The first approach consists of asking voters to compare all possible pairs of candidates successively. The candidate who wins the most head-to-head contests wins the election (there are various ways to handle ties). This method was initially described by Llull and was later proposed by Condorcet. It is now often referred as the **Condorcet method** [2] or **Copeland's method** [3][1]. If one candidate wins all head-to-head contests, she is called the *Condorcet winner.*

In the second approach, voters are asked to rank all candidates, with the lowest-ranked candidate given 1 point, the second-lowest two points, etc. The winner is the candidate with the highest *Borda count*, which is the sum of points received from all voters. This method, called **Borda's method**, was used in the Roman Senate "beginning around 105 AD" [4]; it was proposed by Cusanus to elect Holy Roman Emperors[2] and reinvented by Borda.

Neither of these approaches is without drawbacks. For example, the Condorcet method sometimes does not yield a winner or a consistent ordering of the candidates - this is known as the *Condorcet paradox.* With Borda's method, the withdrawal of even a low-ranked candidate can sometimes change the winner of the election. To overcome such problematic behaviours, many variations of the Condorcet and Borda methods have been devised, including multi-stage methods. None of these voting methods proved to be without flaws.

A very accessible introduction to the problems and paradoxes of the traditional voting systems is given in Powers [5] and in chapter 3 of Balinksi and Laraki [6].

[1] In fact, there are many variations of the Condorcet method, with Copeland's method being the simplest.
[2] His proposal was not adopted.

2.3 Arrow's impossibility theorem

The reason why no perfect solution was ever found in the traditional setting became clear in 1951 when Kenneth Arrow published his famous impossibility theorem. Assume that voters are asked to (strictly) rank at least three candidates. We require a ranking function that ranks all candidates and satisfies the following criteria:

A1. The function produces a ranking for all possible inputs ("unrestricted domain");

A2. if all voters prefer A to B, then A will rank higher than B ("unanimous decision");

A3. the relative rank of A and B does not depend on any other candidate ("independence of irrelevant alternatives", or IIA);

A4. no single voter determines the outcome of the vote ("nondictatorial").

What Arrow proved is that such a function does not exist. For this and other work, he later received a Nobel prize in economics. The usual way of proving Arrow's theorem is to show that any ranking function satisfying A1, A2 and A3 is, in fact, equal to the ranking of one specific voter and ignores the input of all other voters, thus failing condition A4. See Geanakoplos [7] (3rd proof) for a short and extremely elegant proof; Fey [8] also provides a simple proof.

Note that the impossibility holds even without requiring other conditions. For example, one desirable feature of a voting system is "monotonicity": placing a winning candidate higher should never cause that candidate to lose. So if there is a configuration where candidate A is ranked first by the ranking function and one or more voters place that candidate higher, then this should not cause A to be ranked lower. Several instant-runoff methods violate monotonicity. Another desirable feature is resistance to strategic manipulation.

2.4 A note on modern democracies

The inconsistencies observed in various Condorcet and Borda voting systems are quite minor compared to seemingly unfair results seen in democracies that use first-past-the-post voting.

In the 1983 UK general election, the Alliance of the Liberals and SDP received over 25% of the popular vote but won only 3.5% of the seats (23 out of 650) in Parliament. For Labour, 28% of the popular vote translated into 32% of the seats (209 seats), while the Conservatives received 42% of the vote and had an absolute majority of 61% of the seats in Parliament.

In the 2007 presidential election in France, it is considered almost certain that François Bayrou would have won one-to-one against any of the other candidates. But because Bayrou did not advance to the second round, Nicolas Sarkozy became president.

In the 2015 UK general election, Labour added 0.74 million voters but lost 26 seats. The Conservatives gained 24 seats while adding only 0.63 million voters and obtained a slim parliamentary majority. UKIP received 12.6% of the popular vote but ended up with only one of 650 seats in Parliament - a mere 0.15%.

It is interesting, and somewhat puzzling, to note that the Condorcet and Borda methods, whatever their flaws, are often superior to voting methods used in some democratic countries today.

3 From ordinal to cardinal voting

In their book "Majority Judgement", published in 2010, Balinski and Laraki argue that voting theory should be looking beyond traditional methods ([6] p. 56):

> Perhaps the most astonishing aspect of the theory of voting is that despite Arrow's celebrated impossibility theorem and others, the search for a best method has continued

> within the basic framework first conceived by Llull and Cusanus almost a millennium ago (...)

We already know that plurality voting is highly problematic. Ordinal voting, which ranks candidates, is better but also flawed. We need to focus on cardinal voting, which evaluates candidates[3] rather than ranking them. The question is: why has it taken so long for this realisation to sink in?

Arrow explicitly rejected cardinal utility in his famous book, *Social Choice and Individual Values*, which was published in 1951. Only two years later, however, Harsanyi, also a future Nobel prize winner, made a strong case for cardinal utility [9], and in 1955 essentially showed that Arrow's impossibility theorem does not hold when cardinal utility is used [10].

Writing in 2005, Hillinger [11] provides interesting background information and describes how Arrow, as well as others, failed to take much notice of Harsanyi's work for decades – even though Arrow was Harsanyi's thesis advisor! Hillinger concludes his historical perspective by writing:

> Given the long history of voting theory, it is surprising that the first step towards UV[4], in the form of AV[5], was taken as late as the 1970s and the second step, the introduction of a scale with arbitrary divisions, was taken in 2000 with Smith's paper on RV[6].

3.1 The language of voting preferences

A Vote is an expression of one's preference between several candidates or choices. Voting systems impose restrictions on how such preferences can be expressed.

Plurality voting

When a voter can vote for only one candidate, as in plurality voting systems, the expression of preferences is reduced to the bare minimum.

Ordinal voting

In the traditional setting, voters are asked to rank all candidates, which is more expressive than selecting one candidate. This ranking is directly required by Borda, while Condorcet asks for pairwise comparisons. Because the number of pairs increases rapidly with the number of candidates[7], in practice, ranking is most often used to derive the pairwise comparisons.

Unfortunately, the idea of ranking candidates does not allow the voter to express the strength of her preferences. Clearly, the uniform distribution implicit in ordinal voting

[A   B   C   D]

may be quite different from a voter's actual preferences:

[AB   C   D   ]

It also forces voters to compare candidates they know little or nothing about. Finally, the idea of ranking candidates is, in most cases, counter-intuitive. The natural approach is to evaluate candidates independently of each other ([6] p. 113):

> assigning grades is cognitively simple, certainly much simpler than ranking candidates (as any teacher or professor faced with ranking students can attest).

[3]using a so-called cardinal utility function
[4]utilitarian voting
[5]approval voting
[6]range voting
[7]There are $\frac{n(n-1)}{2}$ pairs for n candidates.

Cardinal voting

With cardinal voting, voters give each candidate a grade or rating. When the grades are numeric, the most obvious approach is to add or average the scores obtained by each candidate as described in 3.2.4: this is then variously called *range voting*, *score voting*, *utilitarian voting*, etc.

Because voters' preferences can be quite complex, the choice of scale is important. Assume we are given the following scale, which is the one proposed in Balinksi and Laraki [6]:

Excellent - Very Good - Good - Acceptable - Poor - Reject

A centrist UK voter who is concerned about the environment might rate the Liberal Democrats "excellent", the Greens "very good", both the Conservatives and Labour just "acceptable", give UKIP a "reject" and have no real opinion about the Social Democrats or other minor parties.

If given another scale, say a scale from 0 (worst) to 100 (best), that same voter would be able to express her preferences more precisely, although she might still not know what to do with Sinn Féin, for example: leave it out entirely or decide on some arbitrary number of points representing "I have no opinion". In fact, a 0-100 scale is probably far too wide. In an experiment at École Polytechnique in 2007, students were asked to use such a scale, but 87% of the grades were multiples of 5 (cf [6] p. 306-307).

At the other extreme, we have *approval voting*, which is the simplest form of cardinal voting: it only allows a choice between "approve" and "disapprove". Most voters will find this scale too restrictive.

Note that Likert scales as used in psychometrics typically have between 2 and 10 items, with 3 and 5 being the most common.

Not only the size of a scale but also the scale itself can introduce biases. A recent paper by Baujard et al. [12] analyses the result of an experiment done during the French presidential elections in 2012, where different grade scales were used. One of the paper's conclusions is the following:

> (...) we established that voters use $(-1, 0, 1)$ and $(0, 1, 2)$ in significantly different ways. The $(0, 1, 2)$ scale leads voters to assign a zero either to candidates they know well but reject, and to "unknown" candidates; the scale $(-1, 0, 1)$ often prompts them to award -1 to rejected candidates and the zero grade to unknown candidates who do not inspire the same feeling [of rejection].

Because there seems to be some reluctance to use negative numbers, switching from $(0, 1, 2)$ to the $(-1, 0, 1)$ scale therefore favours unknown candidates. Based on the results of using the scales $(0, 1, 2)$ and $(0, ..., 20)$, the paper also proposes the hypothesis that "evaluative voting with non-negative grades is robust to variations in the precise scale in use."

Early examples of approval voting

Cardinal voting systems were not much studied nor written about until the 20th century, but there are some historical examples of the use of their simplest variant: approval voting.

For example, approval voting was used to elect popes between 1294 and 1621. Venice provides a more interesting example [13]: for over 500 years, between 1268 and 1789, the election of the Doge was a 10-step process. Five of these steps involved drawing by lot. The five other steps, including the last one, involved approval voting with an additional supermajority requirement.

Learning a grading language

The neighbouring countries of Poland and Germany use the same scale of grading from elementary school through university: 1-2-3-4-5-6. There is one minor difference: in Germany, the best grade is 1, while in Poland, it is 6.

The 0-100 US grade system and the 0-20 range used in France appear to be easily comparable: multiply the latter by 5, et voilà! Except that a 12/20 in France means "assez bien" ("fair"), which is well above the 8/20 passing grade in high school[8], while 60/100 in the US rates a borderline D-, just one point above F (fail). The two grading systems are therefore not directly comparable after all.

These differences do not matter much. What matters is that in each of these counties, everyone, teachers, parents and students, has internalised the system and knows what the grades mean. This is essential in any grading system and thus also in any cardinal voting system.

In most countries in the world, the grading scale has 5-6 levels, typically corresponding to a variant of A-B-C-D-(E)-F. Some grading systems have as many as 10 levels, but when wider ranges such as 0-20, 0-60 or 0-100 are used, there is almost invariably also some correspondence to a smaller scale of 5-6 levels. This suggests that it is not necessary to have more than 10 items in a grading language and that 5-6 should suffice in most cases.

3.2 Cardinal voting systems

3.2.1 Definition

Let us define the elements of a cardinal voting system.

- a *language*, which is a strictly ordered set of grades $\Lambda = \{\alpha, \beta, \gamma, ..., \omega\}$ that are not necessarily numerical. We assume the order $\alpha > \beta > \gamma > ... > \omega$
- optionally, a *rating function* r, which translates Λ into a set of numerical ratings R such that $r(\alpha) > r(\beta) > r(\gamma) > ... > r(\omega)$
- a set of n *voters* $V = \{v_i \mid i \in [1..n]\}$
- optionally, a set of n *weights*, $W = \{w_i \mid i \in [1..n]\}$ the sum of which will also be denoted by W; if weights are not necessary, we can simply assume that they are equal to 1, in which case $W = n$
- a set m *candidates* or *choices* $C = \{c_j \mid j \in [1..m]\}$
- a *profile* $\Phi = \{g_{i,j} \mid g_{i,j} \in \Lambda, i \in [1..n], j \in [1..m]\}$, which can be represented as a $m \times n$ matrix in which $g_{i,j}$ represents the grade given by voter i to candidate j; when a rating function exists, we define $r_{i,j} = r(g_{i,j})$
- a *grading function* $F : \Lambda^{m \times n} \rightarrow \Lambda^m$ or $F : \Lambda^{m \times n} \rightarrow \mathbb{R}^m$ whose j-th output value F_j corresponds to a final grade; the outputs of the grading function will often be restricted to an interval, for example [0, 1] or [0, 20]

Voting theory attempts to identify classes of grading functions whose properties make them suitable for use by human societies.

3.2.2 Social grading functions

Balinksi and Laraki [6] consider the standard setting without weights and define a *social grading function (SGF)* as a function that satisfies six axioms:

- *neutrality*: all candidates are treated equally.
- *anonymity*: all voters have the same influence on grades.
- *unanimity*: if all voters give the same grade α to a candidate, then F also gives the same grade.

[8] though a general grade average of 10/20 is needed for promotion

- *monotonicity*: all else being equal, (1) if one or more voters give higher grades to a candidate, the resulting final grade of that candidate cannot be lower; (2) if all voters give a strictly higher grade, the resulting final grade must be strictly higher.
- *independence of irrelevant alternatives ("IIA")*: all else being equal, (1) if one or more voters give higher grades to a candidate, the resulting final grade of that candidate cannot be lower; (2) if all voters give a strictly higher grade, the resulting final grade must be strictly higher.
- *continuity*: very similar lists of grades should be assigned very similar final grades.

These axioms are very logical and represent eminently desirable features of a voting system. When voters can have weights, the neutrality axiom can be restated as follows:

- *anonymity*: the influence on a voter on the final grade is exactly proportional to her weight.

This means that the concept of SGF can easily be extended to situations where voters have different weights. Note that if neutrality holds, a method of grading can always be defined by a single *aggregation function* that operates on each line of the input matrix.

3.2.3 Grading using non-numerical scales and Majority Judgement

If the language is non-numerical and we choose not to use a rating function, the grading function must necessarily be based on counting how often different grades were given to a candidate. As we cannot use means, it seems natural to look at percentiles. In fact, Balinski and Laraki [6] prove that the unique strategy-proof-in-grading[9] social grading functions (as defined above) are the order functions f^k, which simply correspond to the kth highest grade.

Balinsky and Laraki therefore propose to use the median, or lower median in case of an even number of grades. In case of a tie between two sets of grades, the median is removed from both sets and the procedure is applied again until the tie is broken or only one element remains. Their method, *Majority Judgement*, has many desirable properties, including strategy resistance, and avoids Arrow-type paradoxes.

However, this method can yield surprising results. Here is an example from Balinsky and Laraki [6] page 284, in which Y is the Majority Judgement winner despite being rejected by 5 out of 11 voters and despite candidate X being rated "Good" or better by all voters. The reason for this is that X's median grade is "Good", while Y's median grade is "Very Good". In a real-world setting, this result would probably be considered controversial.

	Excellent	Very Good	Good	Acceptable	Poor	To Reject
X:	1	4	6	0	0	0
Y:	0	6	0	0	0	5

Table 1: Controversial "Majority Judgement" winner Y

What this means is that Majority Judgement does not care about dissenting minorities, large though they may be.

3.2.4 Numerical scales

When a rating function is defined, one very natural approach is to compute the normalised sum of grades received by each candidate:

$$F_j = \frac{1}{W} \sum_{i=1}^{n} w_i r_{i,j} \tag{1}$$

[9]If the grade a voter gave to a candidate is above that candidate's final grade, that voter cannot increase the final grade; and if the grade is below the final grade, he cannot reduce it.

When all the weights are equal to 1, we get:

$$F_j = \frac{1}{n} \sum_{i=1}^{n} r_{i,j} \tag{2}$$

Such grading functions also have many desirable properties and meet the SGF criteria. In addition, surprising results such as the one shown in Table 1 would occur less often.

However, as with all methods that use sums, they are vulnerable to strategic manipulation. To mitigate this problem, a proportion of the highest and lowest grades can be removed before summing.

4 Least-Resistance Consensus (LRC)

We now come to the presentation of the Least-Resistance Consensus method, which is a variant of cardinal voting but focuses on the decision-making process itself and the importance of reaching consensus.

4.1 Focus on resistance

The Least-Resistance Consensus (LRC) is based on the idea that it is easier to decide what one is against than what one is in favour of. The sense of rejecting something, our "negative gut feeling", is usually a strong, reliable indicator. In other words, our dislikes are stronger than our likes, and this is probably not just a human characteristic but an evolutionary constant of all living beings. Therefore, LRC aims to only measure the level of rejection, or resistance, and therefore find the common *path of least resistance* of a group.

This approach was partly inspired by the concept of *Via Negativa*, or Apophatic theology, which attempts to approach the Divine in terms of what *cannot* be said about it. The concepts of Via Positiva [14] and Via Negativa [15] are used in theology and denote the two classic ways of approaching the concept of God.

The basic idea underlying LRC is in many ways quite natural; it is therefore not a surprise that others have had similar ideas. For example, the book *Systemisches Konsensieren* by Paulus, Schrotta and Visotschnig [16] promotes ideas that are in essence similar to LRC. Their book was published in 2009; a previous book published in 2005 still used both positive and negative ratings. Their German-language site can be found at `http://www.sk-prinzip.eu/`.

4.2 Grades and lowest average resistance

The LRC concept is extremely simple: instead of measuring the average rating of a choice, we measure the average resistance to it. The candidate or choice with the lowest resistance count wins, but additional conditions may be added to prevent minority dissent.

The languages used are sets of integers starting with 0. Examples are $\{0, 1, 2\}$, $\{0, 1, 2, 3, 5\}$ or $\{0, 1, 2, 3, 4, 5, 6\}$, with the number 0 always denoting the level of least resistance, while the largest number denotes maximal resistance. Different ranges can be used at different stages of a decision process, but it is useful to associate meanings with the resistance points and to encourage voters to specifically think of terms of resistance/objections/opposition to choices.

Table 2 shows possible meanings for the simple $\{0, 1, 2\}$ language. This grading is equivalent to a show-of-hands: no hands up = 0 (agree/mostly agree), one hand up = 1 (not sure or somewhat disagree), two hands up = 2 (disagree). This is such a simple method that even kindergarten-age children should be able to understand and apply it in real-life situations, but it is still significantly more expressive than majority voting.

$\Lambda = \{0, 1, 2\}$

0	I have no, or very few, objections
1	I'm not sure, I feel some resistance
2	I feel definite/strong resistance

Table 2: Show-of-Hands (SoH)

Another, more detailed and expressive grading system is 0-1-2-3-5, which is inspired by the A-B-C-D-F grading system. We therefore call it A/F.

$\Lambda = \{0, 1, 2, 3, 5\}$

0	I have no objections at all
1	I have some minor objections, but nothing very serious
2	I do have some objections and feel some resistance towards this option but still consider it reasonable
3	I feel definite resistance here but could somewhat reluctantly live with it if that's the group's decision
5	I feel strong resistance, and I really don't want this option

Table 3: A/F

In most cases, it will not be necessary to use more complex languages. Here is a simple example inspired by Table 1 using the A/F count and assuming that all voters have the same weight. The winner is candidate X with a significantly lower resistance count than Y.

	0	1	2	3	5	Resistance count
X:	1	4	6	0	0	**16**
Y:	0	6	0	0	5	31

Table 4: Example of a resistance count using A/F

4.3 The LRC decision process

LRC is not simply a voting system – it is designed to support efficient decision-making.

Issue initiation (optional)

In some cases, it will be useful to first determine the list of issues under consideration. Any assembly or group has finite time, and SoH can be used to first determine which issues should be debated.

Compilation of voting items

Once it has been accepted that an issue should be debated and voted on, a decision process starts with stakeholders proposing and discussing possible alternatives. During this exploratory phase, successive voting rounds will identify proposals that meet the least resistance and that will thus be selected for the final voting round. During this phase, SoH can be used.

Voting

During the main voting phase, A/F voting can be used, if necessary with additional safeguards as described below.

4.4 Model

The LRC can be modelled as follows:

- a *language*, which is a strictly ordered set of at least two natural numbers $\Lambda = \{\lambda_1, ..., \lambda_M\}$ such that $\lambda_1 = 0$ and $\lambda_1 < ... < \lambda_M$; these numbers represent a resistance measure
- a set of *ratings* $R = \{r_i \mid i \in [1..M]\} \in [0,1]$ with $r_i = \frac{\lambda_i}{\lambda_M}$; this corresponds to a "renormalisation" of the language to the interval $[0,1]$ and includes both 0 and 1
- a set of n *voters* $V = \{v_i \mid i \in [1..n]\}$
- a set of n *weights*, $W = \{w_i \mid i \in [1..n]\}$; if weights are not necessary, we can simply assume that they are equal to 1, in which case $W = n$
- a set m *candidates* or *choices* $C = \{c_j \mid j \in [1..m]\}$
- a *profile* $\Phi = \{g_{i,j} \mid g_{i,j} \in R, i \in [1..n], j \in [1..m]\}$, which can be represented as a $m \times n$ matrix in which $g_{i,j}$ represents the rating corresponding to the grade given by voter i to candidate j; in the case of abstention, $g_{i,j}$ is set to -1
- in the context of a round of voting, a set $A \subset [1..M]$ represents the indices of the "active" voters who did actually vote; the set $I = [1..M] - A$ represents the "inactive" voters who have abstained from voting; we assume that a voter must vote for all choices or for none

We first define the sum of the weights of all voters and the sum of the weights of those who actually voted in a round:

$$\mathcal{W} = \sum_{i \in [1..n]} w_i \qquad \mathcal{W}^a = \sum_{i \in A} w_i \tag{3}$$

This can be used to express the **Turnout**, rebased to [0,100]:

$$Turnout = 100 * \frac{\mathcal{W}^a}{\mathcal{W}} \tag{4}$$

We can now define an "average resistance count" **ARC** of option j, rebased to [0, 100]:

$$ARC(j) = \frac{100}{\mathcal{W}^a} * \sum_{i \in A} w_i r_{i,j} \tag{5}$$

In order to reduce the impact of strategic manipulation, it may be useful to remove a proportion of the highest and lowest ratings. We call this measure the "truncated average resistance count" **TARC(j, c)** of option j for some $c \in [0, 0.5)$, which can also be denoted $TARC_c(j)$. Note that as c approaches 0.5, the value of TARC will tend towards the median.

Finally, we want to define a function to provide a measure of extreme disagreement. We first define a set of "dissenters" at level $p \in [0,1]$ as the indices of voters who did vote and whose resistance rating was greater than or equal to p:

$$D(j,p) = \{i \in [1..n] \mid g_{i,j} \geq p\} \tag{6}$$

We can now define the **dissenting weight (DW)** of option j at level $p \in [0, 1]$, again rebased to [0,100], as:

$$DW(j,p) = \frac{100}{\mathcal{W}^a} * \sum_{i \in D(j,p)} w_i r_{i,j} \tag{7}$$

4.5 Using LRC in practice

LRC should be seen both as a toolset and a decision-making philosophy. There are no general rules; rather, participants in the decision process should use some of the measures defined above to design an approach that is suitable for their particular group or situation.

That said, in many cases, it will make sense to use ARC or TARC as a first measure, possibly with the restrictions that in order for the vote to be valid, a certain ARC/TARC value (say, 40) should not be exceeded and that a minimum turnout must be reached. In addition, a maximum value (for example, 25) can be put on the dissenting weight at level 0.75. The rule could then be that the option with the lowest ARC/TARC that also has a DW that is too high would be rejected. The winner could be an option for which there is slightly more overall resistance but that is also less divisive.

A few years ago, the author attempted to apply ideas from LRC to designing a decision process in a completely decentralised organisation, the context being the rise in popularity of blockchain platforms and the emerging need for governance on such platforms [17]. Given the hurdles facing the introduction of new voting systems in the political and public areas, it may well be blockchain platforms that will provide large-scale testing grounds for new voting methods.

4.6 Control options and meta-decisions

It is useful to keep certain control items in the list of choices throughout the voting process. Three examples are:

- *ready to decide?* Are we ready to attempt a final vote, or do we need more time to discuss and/or continue looking for alternative options?
- *no-change option*: Sometimes, when the decision is about changing something, group dynamics may cause people to forget that not changing anything is also an option. When a proposed change beats the no-change option only narrowly, maybe it's time to consider the costs and efforts required to make that change.
- *postpone decision*: Not all decisions are urgent, and it can sometimes be useful to postpone a decision.

Sometimes, decisions must be taken quickly, and a lengthy and drawn-out consensus procedure may not be ideal.

- *remove voting restrictions*: Requirements as to the "quality" of consensus and minority veto rules can slow down the decision process. Voters should be able to remove these restrictions when time is of the essence.
- *delegate decision power*: In general, when given enough time and using efficient decision protocols, groups can reach better decisions than individuals. However, when there is no time and emotions run high, a group's decision-making ability can deteriorate. Handing the decision-making power to a single individual or a small group should be an option.

5 Conclusion

We have presented Least-Resistance Consensus, a cardinal voting system that focuses entirely on measuring resistance. Our claim, as yet unproven and unverified, is that measuring the sense of rejection, which is an evolutionary imperative of all living beings, will result in a more efficient and natural consensus-seeking process compared to measuring either only approval or both approval and disapproval.

An additional contribution of this paper is to demonstrate the importance of the decision-making process itself, which is at least as important as the actual voting system.

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